Q.P. Code : 60863

Second Semester M.Sc. Degree Examination, July 2019

(CBCS Scheme)

Mathematics

Paper M 203 T – TOPOLOGY – II

Time : 3 Hours] *[Max. Marks : 70*

Instructions :

1) *Answer any* **FIVE** *full questions.*
2) *All questions carry equal marks.*

1. (a) Define a compact space. Prove that compactness is a topological property.

   (b) Prove that a Hausdorff space is locally compact if and only if each point has a neighbourhood whose closure is compact. **(7 + 7)**

2. (a) Define a Lindelof space. Prove that every second axiom space is a Lindelof space.

   (b) Prove that every separable metric space is second axiom space. **(7 + 7)**

3. (a) Prove that the product space $X_1 \times X_2$ is locally connected if and only if both X_1 and X_2 locally connected.

   (b) Prove that product topology is the smallest topology for which projections are continuous. **(7 + 7)**

4. (a) Define a T_1 - space. Prove that every T_1- space is T_0- space. Give an example of T_0 - space which is not a T_1- space.

   (b) Prove that a topological space (X, τ) is a T_1- space if and only if all singleton sets are closed. **(7 + 7)**

5. (a) Define a Hausdorff space. Show that the property of a space being a Hausdorff space is a hereditary property.

   (b) Prove that a metric space is T_3 - space. **(7 + 7)**

6. (a) Prove that a compact Hausdorff space is normal.

   (b) Prove that a normal space is completely regular if and only if it is regular. (7 + 7)

7. State and prove Urysohn's lemma. (14)

8. (a) Prove that a paracompact Hausdorff space is a normal space.

   (b) Prove that every regular second countable T_1 - space is metrizable. (7 + 7)
